CHOICE OF AVERAGE HEAT LOAD AT THE FUEL ELEMENT SURFACE IN A PRESSURIZED-WATER REACTOR

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A method of determining the maximum permissible average heat load at the fuel element surface in a pressurized-water reactor is proposed for a cosinusoidal distribution of heat release with respect to height and subcooling ≥10° C at the channel outlet.

A limiting factor in the determination of the size of the core of a pressurized-water reactor (PWR) is the critical heat flux. The reactor must be so designed that under all service conditions the heat-release elements operate at subcritical heat loads. Otherwise burnout of the fuel elements and reactor breakdown may ensue.

Since the critical heat flux q_{cr} depends on the subcooling of the heat-transfer agent, q_{cr} decreases with increase in the enthalpy of the latter. If distortion of the neutron flux by the controls is disregarded, the heat release, and hence the heat load q over the height of the fuel element, in a cylindrical core varies in accordance with a cosinusoidal law. Then the maximum value of the heat load usually occurs at the center of the most heavily stressed fuel element. In view of the difference in the laws of variation of the surface heat load and the critical heat flux over the height of the channel (Fig. 1) the question arises: At what point is q_{cr} decisive in relation to the choice of average heat load at the fuel element surface? In the preliminary stages of reactor design it is usual to proceed as follows. The average heat load is determined from the formula

$$q_{av.c} = q_{cr.out}/k_{r\,max} k_{H\,max} k_a k_{s.f_1}, \qquad (1)$$

where

$$k_{r\,max} = q_{r\,max}/q_{av.r}; \quad k_{H\,max} = q_{H\,max}/q_{av.H}; \quad k_a = q_{\kappa\,max}/q_{av.a}.$$

To ensure that the heat load does not exceed the corresponding critical heat flux at any point over the height of the fuel element, the maximum heat load is usually taken less than the minimum critical heat flux [1, 2].

Thus, q_{cr} in Eq. (1) is determined where the heat-transfer agent leaves the fuel element, and the factors in the denominator are related to the center of the most heavily stressed fuel element.

An analysis of expression (1) shows that apart from the necessary safety factor $k_{s.f_1}$ there is a hidden safety factor leading to a reduction in the calculated average heat load at the fuel element surface and hence to an increase in the size of the reactor. This hidden factor is a consequence of the fact that in (1) the numerator and the denominator are determined for different sections of the fuel element, the minimum critical heat flux and the maximum surface heat load being made to coincide, i. e., an unreal case is deliberately assumed for the PWR.

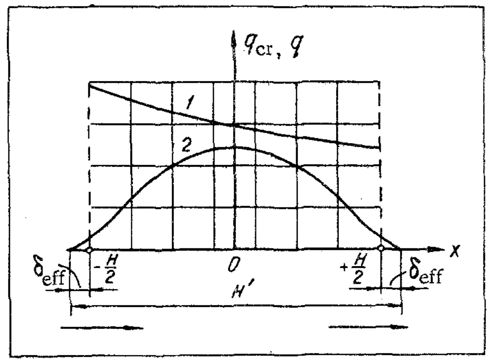

Fig. 1. Variation of 1) critical heat flux q_{cr} and 2) surface heat load $q = q_{H\max} k_a k_r \times \cos(\pi X/H')$ over height of most heavily stressed fuel element (the arrows show the direction of motion of the heat-transfer agent).

In order to exclude the hidden safety factor, it is necessary to determine the average heat load at the point at which the safety factor $k_{s.f}$ is a minimum. In this case we get the maximum possible average heat load and hence the minimum reactor dimensions.

Thus, the maximum possible average heat load at the fuel element surface can be determined from the formula, structurally analogous to (1),

$$q'_{av.c} = q_{cr.X\,min}/k_{r\,max} k_{HX\,min} k_a k_{s.f.X\,min}, \qquad (2)$$

where

$$k_{HX\,min} = \frac{q_H}{q_{av.H}} = q_{H\,max} \cos\frac{\pi X_{min}}{H'} \Big/ \frac{1}{H} \int_{-H/2}^{+H/2} q_{H\,max} \cos\frac{\pi X}{H'} dX = \pi H \cos\frac{\pi X_{min}}{H'} \Big/ 2H' \sin\frac{\pi H}{2H'}.$$

In fact, the ratio of the average heat loads determined from Eqs. (2) and (1) is as follows:

$$\frac{q'_{av.c}}{q_{av.c}} = k_{s.f_1} q_{cr\,X\,min} \Big/ q_{cr.out} \cos\frac{\pi X_{min}}{H'} k_{s.f\,X\,min}.$$

Since the safety factors in (1) and (2) must take into account the inaccuracy of the same design formulas (for determining the heat transfer coefficient and the critical heat flux) and the same mechanical coefficients, $k_{s.f_1} = k_{s.f\,X\,min}$ and the latter ratio takes the form

$$\frac{q'_{av.c}}{q_{av.c}} = q_{cr\,X\,min}/q_{cr.out} \cos\frac{\pi X_{min}}{H'}. \qquad (3)$$

Expression (3) is greater than 1, since

$$q_{cr\,X\,min} > q_{cr.out} \text{ and } \cos\frac{\pi X_{min}}{H'} \leq 1.$$

Thus, other things being equal, using (2) makes it possible to obtain a larger value of the design average surface heat load.

In order to use (2), it is first necessary to find the point at which the safety factor is a minimum. For this purpose, it is sufficient to solve for X an equation of the form

$$dk_{s.f}/dX = 0, \qquad (4)$$

where

$$k_{s.f} = \frac{q_{cr}}{q}, \quad q = k_{r\,max}\, q_{H\,max}\, k_a \cos\frac{\pi X}{H'}.$$

The value $X = X_{min}$, at which (4) vanishes, gives the required point. Obviously, the extreme value of the function $k_{s.f}$ in the interval $(-H/2, +H/2)$ will be its minimum.

In the general case the critical heat flux depends on the subcooling Δt_p, the pressure p, and the mass flow rate γW of heat-transfer agent, as well as on the size and shape of the fuel element cross section d, and may be represented in the form (1)

$$q_{cr} = \text{const}\,(\Delta t_p)^f p^{f_1} (\gamma W)^{f_2} d^{f_3}. \qquad (5)$$

Since the mass flow rate and the diameter of the fuel element do not vary over the height of the latter, while the change in the pressure of the heat-transfer agent can be neglected, the expression for q_{cr} for the fuel element selected may be written as follows:

$$q_{cr} = A\,\Delta t_p^f, \qquad (6)$$

where $A = \text{const}\; p^{f_1}(\gamma W)^{f_2} d^{f_3} = \text{const}$.

We shall determine the change in Δt_p over the height of the fuel element. For this we write the balance equation for heat transfer from the surface of the fuel element to the heat-transfer agent:

$$qsdX = Gcdt. \qquad (7)$$

Substituting $q = q_{Hmax} k_a k_{rmax} \cos \pi X/H'$ and integrating (7) from $-H/2$ to X, we get the change in the temperature of the heat-transfer agent over the height of the fuel element:

$$t = t_{in} + \frac{q_{H\,max} k_a sHk_{r\,max}}{\pi \cdot Gc}\left(\sin\frac{\pi X}{H'} + \sin\frac{\pi H}{2H'}\right). \qquad (8)$$

The subcooling

$$\Delta t_p = t_s - t. \qquad (9)$$

Using (8) we can write the last equation in the form

$$\Delta t_p = t_s - t_{in} =$$

$$= \frac{q_{H\,max} k_a sHk_{r\,max}}{\pi Gc}\left(\sin\frac{\pi X}{H'} + \sin\frac{\pi H}{2H'}\right). \qquad (10)$$

Introducing the subcooling at the outlet of the fuel element channel,

$$\Delta t_{p.out} = t_s - t_{out} = t_s - (t_{in} + \Delta t),$$

substituting for the maximum heat load over the height of the fuel element ($q_{H\,max}$) the average heat load ($q_{av.\,H}$)

$$q_{H\,max} = q_{av.\,H} H\pi \Big/ 2H' \sin\frac{\pi H}{2H'} \qquad (11)$$

and taking into account that $\Delta t = q_{av,\,H} sHk_a k_{r\,max} / cG$, we can write Eq. (10) in the form

$$\Delta t_p = \Delta t_{p.out} + \frac{\Delta t}{2}\left(1 - \sin\frac{\pi X}{H'} \Big/ \sin\frac{\pi H}{2H'}\right). \qquad (12)$$

Substituting (12) into (6), we get the law of variation of the critical heat flux over the height of the fuel element,

$$q_{cr} = A\left[\Delta t_{p.out} + \frac{\Delta t}{2}\left(1 - \sin\frac{\pi X}{H'} \Big/ \sin\frac{\pi H}{2H'}\right)\right]^f. \qquad (13)$$

Using (13) and (11), we can write Eq. (4) for $X = X_{min}$ as follows:

$$\frac{d}{dX}\left\{A\left[\Delta t_{p.out} + \frac{\Delta t}{2}\left(1 - \sin\frac{\pi X_{min}}{H'} \Big/ \sin\frac{\pi H}{2H'}\right)\right]^f \cdot 2H' \sin\frac{\pi H}{2H'}\right\} \times$$

$$\times \left(q_{av.H} H \pi \cos\frac{\pi X_{min}}{2H'}\right)^{-1} = 0. \qquad (14)$$

We shall introduce the notation

$$M = \Delta t_{p.out} + \frac{\Delta t}{2}\left(1 - \sin\frac{\pi X_{min}}{H'} \Big/ \sin\frac{\pi H}{2H'}\right),$$

$$B = 2H' \sin\frac{\pi H}{2H'} \Big/ q_{av,\,H}\,\pi H.$$

Then (14) takes the form

$$\frac{d}{dX}\left(AM^f B \Big/ \cos\frac{\pi X_{min}}{H'}\right) = 0. \qquad (15)$$

Differentiating (15) and making simplifying transformations, we get

$$M^f\left(\sin\frac{\pi X_{min}}{H'} \Big/ \cos^2\frac{\pi X_{min}}{H'} - \right.$$

$$\left. - f\Delta t \Big/ 2M \sin\frac{\pi H}{2H'}\right) = 0. \qquad (16)$$

Since $M^f > 0$, for (16) to be satisfied it is necessary that

$$\sin\frac{\pi X_{min}}{H'} \Big/ \cos^2\frac{\pi X_{min}}{H'} = f\Delta t \Big/ 2M \sin\frac{\pi H}{2H'}. \qquad (17)$$

Expression (17) leads to a quadratic equation, whose solution is

$$\sin\frac{\pi X_{\min}}{H'}=\left[\sin\frac{\pi H}{2H'}\left(1+\frac{2\Delta t_{\text{p.out}}}{\Delta t}\right)\pm\right.$$
$$\left.\pm\sqrt{\sin^2\frac{\pi H}{2H'}\left(1+\frac{2\Delta t_{\text{p.out}}}{\Delta t}\right)^2-4f(1-f)}\ \right]\times$$
$$\times\,[2(1-f)]^{-1}. \qquad (18)$$

Analysis of the last expression for $\Delta t_{\text{p.out}} > 0$ shows that the plus sign in front of the root does not make sense, since in that case the right side becomes greater than unity.

Keeping the minus sign in front of the root, we find the point at which the safety factor is a minimum,

$$X_{\min}=\frac{H'}{\pi}\arcsin\left\{\left[\sin\frac{\pi H}{2H'}\left(1+\frac{2\Delta t_{\text{p.out}}}{\Delta t}\right)-\right.\right.$$
$$\left.-\sqrt{\sin^2\frac{\pi H}{2H'}\left(1+\frac{2\Delta t_{\text{p.out}}}{\Delta t}\right)^2-4f(1-f)}\ \right]\times$$
$$\left.\times\left[2(1-f)\right]^{-1}\right\}. \qquad (19)$$

From (19) it follows that the coordinate of the point of minimal safety factor $X_{\min}$ depends, in the general case, on the height of the fuel element, the reflector savings, the degree of heating of the heat-transfer agent in the most heavily stressed fuel element and the subcooling at its outlet, as well as on the exponent of the subcooling in the formula for q_{cr}.* All these characteristics, except the first two are known. The height of the fuel element and the reflector savings in the stage in question are not known. Therefore, in order to avoid successive approximations, Eq. (19) can be replaced with an approximate formula based on the following considerations. In most cases of practical interest (e.g., naval reactors) the height of the fuel elements in a PWR varies in the range 1–2 m, the reflector savings are equal to 0.08–0.1 m. Thus, the ratio H/H' lies in the range $0.835 \le H/H' \le 0.925$. If we determine $X_{\min}$ for mean values of H/H' and δ_{eff}, i.e., $H/H' = 0.88$ and $\delta_{\text{eff}} = 0.09$ m, then (19) can be written in the form

$$X_{\min}\approx 0.477\arcsin\left\{\left[0.982\left(1+\frac{2\Delta t_{\text{p.out}}}{\Delta t}\right)-\right.\right.$$
$$\left.-\sqrt{0.965\left(1+\frac{2\Delta t_{\text{p.out}}}{\Delta t}\right)^2-4f(1-f)}\ \right]\times$$
$$\left.\times\,[2(1-f)]^{-1}\right\}. \qquad (20)$$

*At present, for determining the critical heat fluxes at the pressure of the primary loop $137\cdot10^5$–$206\cdot10^5$ N/m^2 and subcoolings from 100° to 10° C widespread use is being made of the Zenkevich-Subbotin formula [3], in which $f = 0.33$; at subcoolings >50° C and pressures of $73\cdot5\cdot10^5$–$147\cdot10^5$ N/m^2 Ornatskii and Kichigin [4] recommend $f = 0.6$.

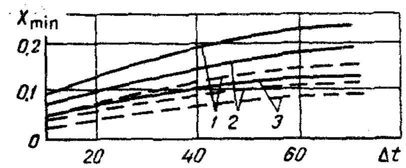

Fig. 2. Variation of coordinate $X_{\min}$ for $f = 0.33$: 1) when $H/H' = 0.86$; 2) 0.905; 3) 0.925; the continuous lines are for $\Delta t_{\text{p.out}} = 10°$ C, the broken lines for 25° C.

For the approximate value of $X_{\min}$ obtained from (20) for given $\Delta t_{\text{p.out}}$ (not less than 10° C) and Δt and selected f, we can determine the critical heat flux corresponding to this point and the coefficient of nonuniformity of heat release over the height of the fuel element,

$$q_{\text{cr.}\,X\min}\approx A\left[\Delta t_{\text{p.out}}+\frac{\Delta t}{2}\left(1-\frac{\sin 2.090\,X_{\min}}{0.982}\right)\right]^f, \qquad (21)$$

$$k_{HX\min}\approx 1.410\cos 2.090\,X_{\min}. \qquad (22)$$

Taking the coefficients of nonuniformity of heat release over the radius of the core and over the fuel assembly on the basis of existing experience with the design of reactors, determining (with allowance for the inaccuracy of the design formulas for the critical heat flux, the heat transfer coefficient, etc.) the safety factor from Eq. (2) and using (21), (22), we can find the maximum permissible average surface heat load. The value obtained differs by no more than ±4% in absolute magnitude from that computed for the exact $X_{\min}$ [(19)]. In this case for small H/H' the calculated value will be too small as compared with the exact computation (for $H/H' = 0.835$ by up to −4%), and for large H/H' too big (at $H/H' = 0.925$ by up to +4%). Thus, when the ratio H/H' is not known in advance, the maximum permissible heat load determined from (2) with account for (20), (21), and (22) should be reduced by 4%.

It is desirable to determine the increase in the average heat load as found from (2) in relation to that obtained by the usual method [(1)]. For this we turn to expression (3), which we expand, bearing in mind that

$$q_{\text{cr.}\,X\min}=\frac{q_{\text{cr.}\,0}\,\Delta t^f_{\text{p.}\,X\min}}{\Delta t^f_{\text{p.0}}}=q_{\text{cr.}\,0}\left[\Delta t_{\text{p.out}}+\right.$$
$$\left.+\frac{\Delta t}{2}\left(1-\sin\frac{\pi_{X\min}}{H'}\Big/\sin\frac{\pi H}{2H'}\right)\right]^f\times$$
$$\times\left[\left(\Delta t_{\text{p.out}}+\frac{\Delta t}{2}\right)^f\right]^{-1},$$

$$q_{\text{cr.out}}=\frac{q_{\text{cr.0}}\,\Delta t^f_{\text{p.out}}}{\Delta t^f_{\text{p.0}}}=\frac{q_{\text{cr.0}}\,\Delta t^f_{\text{p.out}}}{(\Delta t_{\text{p.out}}+\Delta t/2)^f}.$$

Then we can write

$$\frac{q'_{\text{av.c}}}{q_{\text{av.c}}}=\left[\Delta t_{\text{p.out}}+\frac{\Delta t}{2}\left(1-\sin\frac{\pi X_{\min}}{H'}\Big/\sin\frac{\pi H}{2H'}\right)\right]^f\times$$

$$\times \left(\Delta t^{i}_{p.out} \cos \frac{\pi X_{min}}{H'}\right)^{-1}.$$

When $f = 0.33\ X_{min}$ is close to 0 (Fig. 2); therefore for estimating the value of the ratio we can set $X_{min} = 0$. Then

$$q'_{av.c}/q_{av.c} \approx (1 + \Delta t/2\Delta t_{p.out})^{0.33}. \qquad (23)$$

It follows from (23) that the average heat load determined from (2) will exceed the value determined from (1) by an amount that is greater, the greater the heating of the heat-transfer agent in the reactor and the smaller the subcooling at the fuel element outlet. Thus, for $\Delta t = 50°$ C and $\Delta t_{p.out} = 10°$ C it will be roughly 50% higher, for $\Delta t = 50°$ C and $\Delta t_{p.out} = 25°$ C roughly 25% higher.

In conclusion, it is desirable to examine the case, common for PWR, when the subcooling at the outlet of the most heavily stressed fuel element is selected so that for all possible deviations from rated operating conditions the condition $\Delta t_{p.out} \geq 0$ is satisfied. This requirement is satisfied when under design conditions $\Delta t_{p.out}$ is $(0.3-0.4)\Delta t$. In this case (23) is written in the form

$$q'_{av.c}/q_{av.c} \approx [1 + \Delta t/(0.6-0.8)\Delta t]^{0.33} \approx 1.3.$$

Thus, using (2), we get an increase in the calculated average surface heat load of ≈30%.

NOTATION

q—heat load at surface of fuel element; $q'_{av.c}$, $q_{av.c}$—average surface heat load; q_{cr}—critical heat flux; k—coefficient of nonuniformity of heat release; k_{sf}—safety factor [k_{sf_1}—safety factor in Eq. (1)]; H, d, s—height, diameter, and perimeter of fuel element; p, γW, Δt_p, t_s, c, t—respectively, pressure, mass flowrate, subcooling, saturation temperature, heat capacity, and temperature of heat-transfer agent in primary loop; G, Δt—flow rate and heating of heat-transfer agent in most heavily stressed fuel element; dt—increase in temperature of heat-transfer agent on height dx; f, f_1, f_2, f_3—exponents in the expression for q_{cr}. Subscripts: r, H—radius and height of core; c—core; a—fuel assembly; X_{min}—point along fuel element at which k_{sf} is a minimum; out, in—outlet and inlet of heat-transfer agent; max—maximum value; 0—center of fuel element.

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